NAME:

Math 150 Practice Exam 1.2

Instructions: WRITE YOUR NAME CLEARLY. Do as many problems as you can for a maximal score of 100. SHOW YOUR WORK!

1. Calculate $\lim_{x \to 9} \frac{\sqrt{x}-3}{x-9}$ [10 pts]

2. Evaluate $\lim_{x \to 3} \frac{(x-1)(x-2)}{(x-3)}$ or explain why this limit doesn't exist.

[10 pts]

3.   Evaluate $\lim_{x \to -\infty} 4x\left(3x - \sqrt{9x^2 + 1}\right)$ [10 pts]

4.   Compute $\lim_{x \to 0} \frac{\cos x - 1}{\sin^2 x}$ or explain why the limit doesn't exist.

[10 pts]

5.   a) Find the derivative of $f(x) = 5x^2 - 6x + 1$ using the definition of the
derivative at the point $a = 2$.   [5 pts]

b) Use the result in part (a) to write an equation of the line at the point
$(a, f(a))$ [5 pt]

6.   Evaluate $\displaystyle \lim_{x \to 0} \frac{\tan 5x}{x}$ [10 pts]

7.   Show that the equation $x^3 - 5x^2 + 2x = -1$ has a solution.   [10 pts]

8.   Let $a > 0$ be a positive real number. Define $f(x) = \begin{cases} \sqrt{2x} & \text{if } x < a \\ x & \text{if } x \geq a \end{cases}$.

What is the value of a if f is continuous on the entire real number line?   [10 pts]

9.   Compute $\lim_{x \to 0} \frac{\sqrt{1+5x} - \sqrt{1-5x}}{x}$ or explain why the limit doesn't exist.

[10 pts]

10. Compute the derivative of $f(x) = \dfrac{(x^2-1)\sin x}{\sin x + 1}$ [10 pts]

Extra-Credit

11. Prove by means of a delta-epsilon argument that if $\lim_{x \to a} f(x) = L$ and $\lim_{x \to a} g(x) = M$ then $\lim_{x \to a} (f(x) + g(x)) = L + M$

[10 pts]

12. Prove from the definition of the derivative that $\frac{d}{dx}\left(x^{1/n}\right) = \frac{1}{n}x^{\frac{1}{n}-1}$

[10 pts]